

A Simple Strategy to Teach Almost Any Math Skill:

The Concrete-Representational-Abstract
Approach

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Introduction to the Concrete-Representational-Abstract (CRA) Approach

- › Take more abstract concepts and demonstrate them with concrete objects and pictures.
- › Gives an understanding of the core concepts behind math problems (Witzel & Little, 2016).
- › Can help close gaps in mathematics knowledge (Allsopp et al., 2008).

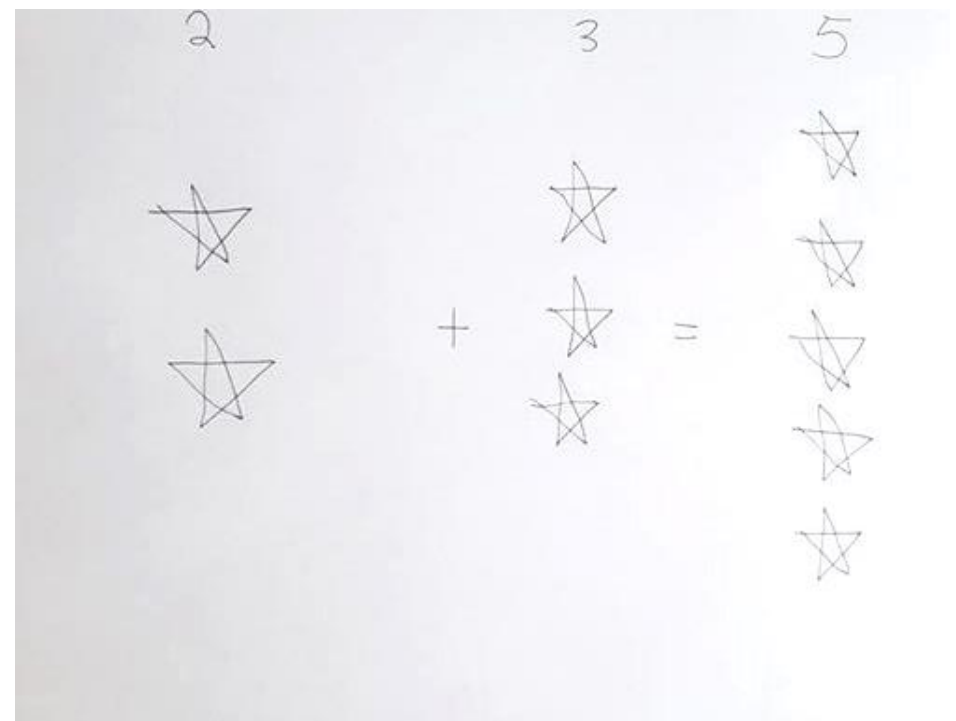
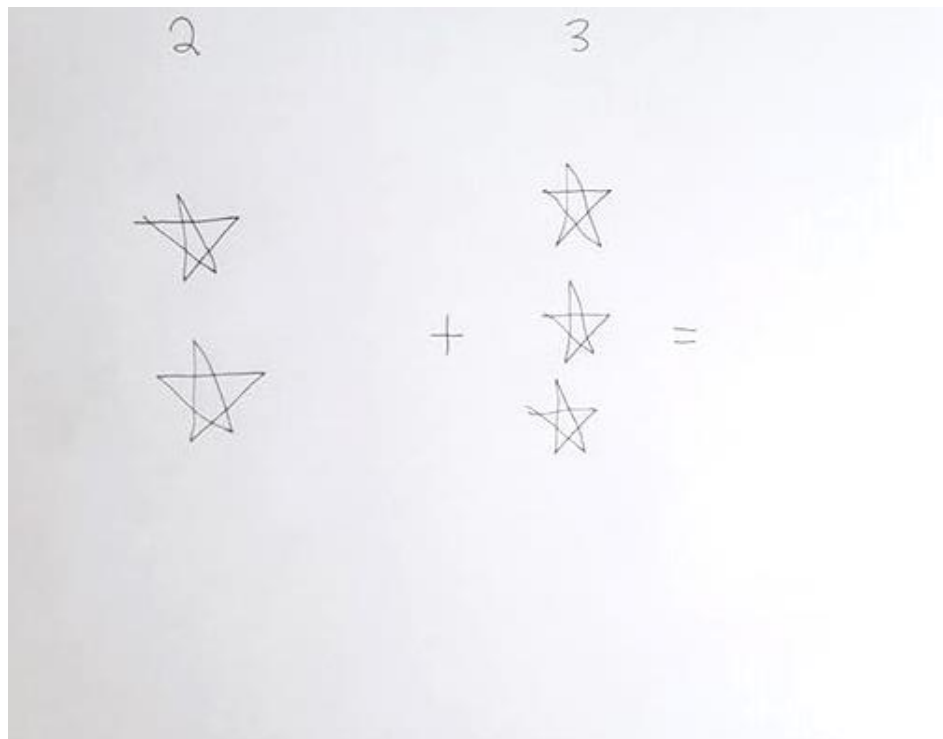
Three Stages in the CRA Approach

1. Concrete: demonstrate with physical objects, such as blocks or toys (Flores, 2010).



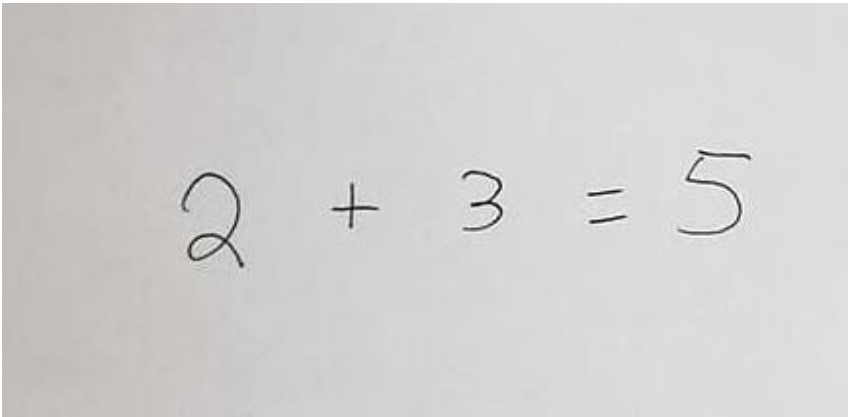
Three Stages in the CRA Approach

2. Representational: objects are replaced by drawings (Flores, 2010).



Three Stages in the CRA Approach

3. Abstract: students solve problems using written numbers only
(Miller et al., 2011).

A photograph of a piece of paper with the equation $2 + 3 = 5$ written in black ink. The numbers and symbols are handwritten and slightly irregular, typical of a student's work.
$$2 + 3 = 5$$

To Make It Even More Fun!

- › You can use the student's interests as a teaching tool.
- › Favorite characters, figures, toys, etc. can be used during the concrete and representational stages (drawings or digital images for the representational stage).



Teaching Procedures at Each Stage

At each stage, you can follow these steps, either with objects, drawings, or numbers.

1. The teacher demonstrates the math concepts and models solving problems while the student watches.
2. The student works on problems with prompts, cues, and guidance from the teacher.
3. The student works on problems independently.

As a note, when teaching at the abstract level, you may also add a mnemonic strategy to help students memorize how to solve the problem (Flores, 2010), such as remembering the order of operations with "Please Excuse My Dear Aunt Sally."

Research Support

Can be used to teach various skills from basic math (such as adding and subtracting) to more advanced concepts such as algebra.

It includes various instructional components that are considered effective, such as:

- › Visual supports,
- › Many opportunities for teacher feedback,
- › Guided and independent practice,
- › Systematic and direct instruction (Witzel & Little, 2016),
- › Explicit instruction (teaching that is clear/unambiguous) (Miller et al., 2011).

Tips

- › Make sure that you are using the same vocabulary and procedures consistently across each stage (Witzel & Little, 2016).
- › To learn more about teaching with this approach, check out the articles and books in the reference list at the end of this handout.

Examples: Addition and Subtraction

The following links have some great picture examples of this approach.

- › <http://ebi.missouri.edu/wp-content/uploads/2013/08/EBI-Brief-Template-Concrete-Representational-Abstract.pdf>
- › <https://www.pinterest.com/pin/93731235982778025/>
- › <https://www.pinterest.com/pin/559501953682141682/>
- › <https://www.pinterest.com/pin/559501953682141682/>

Examples: Multiplication and Division

The following links have some great picture examples of this approach.

- › <http://ebi.missouri.edu/wp-content/uploads/2013/08/EBI-Brief-Template-Concrete-Representational-Abstract.pdf>
- › <http://fcit.usf.edu/mathvids/strategies/images/MultDiv1.jpg>
- › <http://fcit.usf.edu/mathvids/strategies/images/MultDiv2.jpg>
- › <http://fcit.usf.edu/mathvids/strategies/images/MultDiv3.jpg>

Examples: Place Value

The following links have some great picture examples of this approach.

- › <https://www.pinterest.com/pin/81205599515985469/>
- › <http://fcit.usf.edu/mathvids/strategies/images/Place5.jpg>
- › <http://fcit.usf.edu/mathvids/strategies/images/Place7.jpg>

Examples: Fractions

The following links have some great picture examples of this approach.

- › <https://www.pinterest.com/pin/552324341799374151/>
- › <http://fcit.usf.edu/mathvids/strategies/images/Fract1.jpg>
- › <http://fcit.usf.edu/mathvids/strategies/images/Fract2.jpg>
- › <http://fcit.usf.edu/mathvids/strategies/images/Fract3.jpg>

Other Examples

The following links have some great picture examples of this approach.

- › Positive and Negative Integers (one color represents positive and the other color represents negative and the “cancel each other out”):
<http://fcit.usf.edu/mathvids/strategies/images/PosNeg1.jpg>
- › Geometry: <http://fcit.usf.edu/mathvids/strategies/images/Geo1.jpg>
- › Geometry: <https://amzn.to/2VBAdB4>
- › Algebra: <http://fcit.usf.edu/mathvids/strategies/images/Alg1.jpg>
- › Algebra (goes along with the first photo):
<http://fcit.usf.edu/mathvids/strategies/images/Alg2.jpg>

Resources for More Information

Book: “[Teaching Elementary Mathematics to Struggling Learners](#)”
by Bradley S. Witzel and Mary E. Little

Blog post by Dr. Caldwell: <https://www.autismhomeschoolsuccess.com/single-post/2019/06/29/Math-Teaching-Tip-Build-a-Bridge-from-Concrete-to-Abstract-with-the-Concrete%E2%80%93Representational%E2%80%93Abstract-Approach>

Articles

- › Flores, M. M., Hinton, V. M., Strozier, S. D., & Terry, S. L. (2014). Using the concrete-representational-abstract sequence and the strategic instruction model to teach computation to students with autism spectrum disorders and developmental disabilities. *Education and Training in Autism and Developmental Disabilities, 49*(4), 547-554.
- › Flores, M. M., & Hinton, V. M. (2019). Improvement in elementary students' multiplication skills and understanding after learning through the combination of the concrete-representational-abstract sequence and strategic instruction. *Education and Treatment of Children, 42*(1), 73-100.
- › Yakubova, G., Hughes, E. M., & Shinaberry, M. (2016). Learning with technology: Video modeling with concrete-representational-abstract sequencing for students with autism spectrum disorder. *Journal of Autism and Developmental Disorders, 46*, 2349-2362.

References

Allsopp, D. H., Kyger, M. M., Lovin, L., Gerretson, H., Carson, K. L., Ray, S. (2008). Mathematics dynamic assessment: Informal assessment that responds to the needs of struggling learners in mathematics. *Teaching Exceptional Children, 40*(3), 6-16.

Bouck, E., Park, J., & Nickell, B. (2017). Using the concrete-representational-abstract approach to support students with intellectual disability to solve change-making problems. *Research in Developmental Disabilities, 60*, 24-36.

Flores, M. M. (2010). Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure. *Remedial and Special Education, 31*(3), 195-207.

Miller, S. P., Stringfellow, J. L., Kaffar, B. J., Ferreria, D., & Mancl, D. B. (2011). Developing Computation Competence Among Students Who Struggle with Mathematics. *Teaching Exceptional Children, 44*(2), 38-46.

Witzel, B. S., & Little, M. E. (2016). *Teaching elementary mathematics to struggling learners*. New York, NY: The Guilford Press